Abstract: todo

Keywords: Knapsack,

# Introduction

Imagine a daring thief who has found himself amidst a legendary treasure trove, equipped with just a worn knapsack on his back. As his eyes dart across the most enticing items in the room, he is sorely reminded that his bag cannot withstand the weight of everything he wants. He finds himself forced to make strategic decisions - Should he go for the priceless artifact, the rare antique watch, or the ornate silver vase? They each have different weights and values. Imposed by the weight limit of his bag, the thief finds himself facing the same challenge as some of the greatest intellects for generations.

This modified version of the classic Thief example from Bhargava demonstrates an intriguing optimization challenge, known as the Knapsack Problem (KP), that has tested mathematicians, computer scientists, and even fictional thieves throughout history. At its core, the KP focuses on optimal resource allocation by framing it into one basic question: How can we maximize the value of the items in the knapsack without exceeding a weight limit?

# Problem Statement

## *Sub Heading NEEDED*

The classic KP challenge revolves around the efficient packing of items into a knapsack with a limited weight capacity. The problem includes a knapsack with a predetermined weight capacity and a variety of items, each characterized by its corresponding weight and value. It is assumed that the weight capacity is at least as large as the heaviest item, and as many items as possible can be stored in the bag until the weight capacity is me. As the challenge focuses on distribution of the items, real world constraints such as dimensions and size are ignored [1].

While there are many variations of the KP, one of the most discussed and researched is known as the 0/1 KP. Some variations allow for fractions of items to be added to the bag in order to obtain the most value, while 0/1 KP asserts that each item can be either be entirely included in the knapsack or entirely excluded. It is important to note that while we often allude to items going in a knapsack, mathematically, this problem simplifies down to simply a "combination of binary decisions" [6]. For each item in the knapsack, it is either in the bag (1) or not in the bag (0). The duration of this report will be in regards to the 0/1 KP, hereto referred to simply as KP, unless otherwise specified.

## *Mathematical Representation*

In the formal representation of the KP, a combination of a given set of items, characterized by their corresponding weight () and value (), are selected to place in the knapsack, such that the value is maximized while the weight does not exceed the knapsack's capacity (). This representation can be expressed as follows:

(1)

where

* represents the profit/value of item
* corresponds to the weight of item
* denotes the knapsack's maximum weight capacity
* is a binary decision variable that takes the value if item is included in the knapsack and otherwise.

The function is used to represent the total value/profit achieved by selecting items with their corresponding profits () .The constraint ensures that the total weight of the selected items does not exceed the knapsack's capacity () by restricting the combination of items that can be placed in the knapsack. Since this is the 1/0 variation of the knapsack problem, is used to represent the selection () or exclusion () of items, enabling the optimization process to find the optimal combination of items that maximizes the total profit while adhering to the weight constraint [6].

## *History*

The KP has a long history, dating back centuries to its first appearance in 1895 [6]. Although the official name had yet existed at the time, Mathews' work on Partition Theory introduced the underlying connections to combinatorial optimization problems like the Knapsack problem. For example, Partition Theory’s focus on dividing or representing a number as a sum of positive integers introduced the notion of dividing the knapsack's capacity into parts (weights) to maximize the overall value. This was just the beginning and set the stage for future developments in the KP [6][4]. Combinatorial optimization is a branch of mathematics which deals with finding the best solution from a limited selection and a finite set of possible combinations. However, the KP is just one of many problems in this category; Other well-known combinatorial optimization problems include the Traveling Salesman Problem, the vehicle routing problem, and the Graph Coloring Problem, to name a few [11].

The introduction of the 0/1 variant of the KP in 1980 by Gallo, Hammer, and Simeone further expanded the problem's scope. The addition of the decision variables () brought an additional layer of complexity to the KP which opened new avenues for research and algorithmic exploration [9]. Since then, researchers have been working to improve the algorithm efficiency to various instances of the KP and its variants. Through which significant algorithmic advancements have been made, enabling the solution of “nearly all standard instances [of the KP] found in the literature” [7].

## *Importance*

The importance of the KP serves as a powerful tool across the fields of mathematics and computer science, whether to explore how algorithms work or how efficiently they operate [1], or in day-to-day practical applications, showcasing its versatility and utility in solving diverse challenges. Computer scientists can use the KP to help in program partitioning and task allocation, where it is important to prioritize critical tasks while optimizing computational resources. For more a more practical example, the KP can be used to help efficiently pack food for survival situations, with a goal to maximize nutrition while minimizing overall weight [5]. Additionally, the KP can help assist investors in maximizing their profits while adhering to their budget constraints. The KP proves invaluable to the lumber industry, where the KP contributes to the optimization of log cutting processes, maximizing the value obtained from each log while minimizing waste. In fact, while KP is often considered a ‘packing’ problem, it also sometimes referred to as a ‘cutting problem [6].

The versatility of the Knapsack is evident in many real-world applications, from efficiently packing food for survival to optimizing investment portfolios, loading cargo planes, and even cutting logs, the real world often presents additional complexities that the basic KP does not account for. From nutritional sustainability, investment risk tolerance, time constraints, market price variations, to other practical factors, there appeared a substantial “need for [an] extension of the basic knapsack model,” which led to the development of “various extensions and variations,” with each variant offering a solution to tackle a specific scenario [6]. These extensions further enhance the KP’s applicability and make it invaluable across various industries and applications.

## Hurbans

* + Grokking Artificial Intelligence Algorithms
  + Rishal Hurbans
  + https://learning.oreilly.com/library/view/grokking-artificial-intelligence/9781617296185/

## Bhargava

* + Grokking Algorithms
  + Aditya Bhargava
  + <https://learning.oreilly.com/library/view/grokking-algorithms/9781617292231/>

## Geeks

* + <https://www.geeksforgeeks.org/0-1-knapsack-problem-dp-10/>

## Mathews

* + Mathews, G. B. (25 June 1897). "On the partition of numbers" (PDF). Proceedings of the London Mathematical Society. 28: 486–490. doi:10.1112/plms/s1-28.1.486.
  + <https://books.google.com/books?id=DlJAAQAAIAAJ&printsec=frontcover&source=gbs_ge_summary_r&cad=0#v=onepage&q&f=false>

## Rocca

* + Advanced Algorithms and Data Structures
  + By Marcello La Rocca
  + <https://learning.oreilly.com/library/view/advanced-algorithms-and/9781617295485/>

## Kellerer

* + Knapsack Problems
  + Hans Kellerer , Ulrich Pferschy , David Pisinger
  + <https://link.springer.com/book/10.1007/978-3-540-24777-7>

## Pisinger

* + Where are the hard knapsack problems?
  + David Pisinger
  + <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.87.7431&rep=rep1&type=pdf>

## Zingaro

* + Algorithmic Thinking
  + Dan Zingaro
  + <https://learning.oreilly.com/library/view/algorithmic-thinking/9781098128197/>

## Billionnet

* + Linear programming for the 0-1 quadratic knapsack problem
  + Alain Billionnet
  + On drive
  + <https://www.sciencedirect.com/science/article/pii/0377221794002290>

## Andonov

* + Unbounded knapsack problem: Dynamic programming revisited
  + <https://www.sciencedirect.com/science/article/pii/S0377221799002659>

## Wang

* + https://www.sciencedirect.com/science/article/abs/pii/S0020025522013627